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PARAMETRIC DESIGN CURVES FOR SHORT RANGE, HIGH ACCELERATION BALLISTIC ROCKETS

1 April 1963

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PARAMETRIC DESIGN CURVES FOR SHORT RANGE, HIGH ACCELERATION BALLISTIC ROCKETS

by

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Department of Army Project No. 1-A-2-22901-A-202

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Future Missiles Systems Division
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Redstone Arsenal, Alabama

ABSTRACT

This report is a compilation of design curves which will permit the rapid estimation of the effects of range, warhead weight, diameter, specific impulse, and propellant weight fraction on the weight and length of short range, high acceleration ballistic rockets.

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LIST OF SYMBOLS

Symbol	Units	Definition
C	lb/in.^{2}	Ballistic coefficient of the rocket at burnout $\frac{W_{bo}}{id^2}$
d	in.	Rocket diameter
g	ft/sec^2	Gravity constant
I_{sp}	sec	Propellant specific impulse
i		Drag form factor related to BRL type 2 projectile, $i = \frac{\text{drag}}{\text{drag of type 2}}$
k_1, k_2		Constants in curve fit of range tables
L_D		Volumetric loading fraction of motor
l	in.	Length
PWF		Propellant weight fraction, $\frac{W_{prop}}{W_{to} - W_{wh}}$
ρ_{wh}	lb/in.^{3}	Average warhead density
ρ_P	lb/in.^{3}	Propellant density
Q		Growth factor, $\frac{W_{to}}{W_{wh}}$
Q. E.	deg	Quadrant elevation or launch angle
γ_B		Booster mass ratio, $\frac{W_{to}}{W_{bo}}$
V_{actual}	ft/sec	Boost velocity including effects of drag and gravity

LIST OF SYMBOLS (Concluded

Symbol	Units	Definition
V_{ideal}	ft/sec	Boost velocity neglecting drag and gravity
W_{bo}	lb	Rocket weight at booster burnout
W_{prop}	lb	Propellant weight
W_{to}	lb	Rocket weight at takeoff
W_{wh}	lb	Weight of warhead section

PARAMETRIC DESIGN CURVES FOR SHORT RANGE, HIGH ACCELERATION BALLISTIC ROCKETS

I. INTRODUCTION

The preliminary design of a rocket to satisfy a given set of performance requirements usually involves laborious calculations of the effects of various design parameters on the physical characteristics of the rocket. The data presented in this report will allow the rapid estimation of the effects of range, specific impulse, propellant weight fraction, payload weight, and diameter on the weight and length of short range, high acceleration ballistic rockets.

II. SCOPE

The data presented in this report cover the following ranges of parameters:

Range	10 to 40 kilometers
Specific impulse	200 to 260 seconds
Propellant weight fraction	0.5 to 0.9
Payload ballistic factor, W_{wh}/id^2	0.5 to 2.0

III. ASSUMPTIONS

The boost acceleration is assumed to be high enough so that the burning distance is very small compared to the range of the rocket. This allows the use of the Ballistics Research Laboratories range tables*, which are based on zero burning distance.

Exterior Ballistics Tables for Projectile Type 2, BRL Memorandum Report No. 1096, Aug. 1957.

Boost phase drag and gravity velocity loss are assumed to be five per cent of the ideal velocity given by Equation 1.

$$V_{ideal} = I_{sp}\, g \ln\left(\frac{W_{to}}{W_{bo}}\right). \tag{1}$$

The actual burnout velocity is therefore given as

$$V_{actual} = 0.95\, I_{sp}\, g \ln\left(\frac{W_{to}}{W_{bo}}\right). \tag{2}$$

IV. APPLICABLE EQUATIONS

A. Booster Mass Ratio

The booster mass ratio ($W_{to}/W_{bo} = \gamma_B$) required for a given velocity is obtained from Equation 2 as

$$\gamma_{req'd} = e^{(V_{B\ req'd}/0.95\, I_{sp}\, g)} \tag{3}$$

B. Growth Factor

The growth factor of the rocket (ratio of takeoff weight to warhead weight) is derived from the definition of takeoff weight given in Equation 4,

$$W_{to} = W_{wh} + \frac{W_{prop}}{PWF}, \tag{4}$$

and the definition of burnout weight given in Equation 5,

$$W_{bo} = W_{wh} + W_{prop}\left(\frac{1}{PWF} - 1\right). \tag{5}$$

The growth factor is given as

$$\frac{W_{to}}{W_{wh}} = \frac{\gamma_B \times PWF}{\gamma_B \times PWF - \gamma_B + 1.0}. \tag{6}$$

C. Ballistic Coefficient

The ballistic coefficient used in this report is defined as

$$C = \frac{W_{bo}}{id^2} , \tag{7}$$

where i is the ratio of the drag coefficient of the rocket under consideration to the drag coefficient on which the range tables are based (BRL Type 2).

The ballistic coefficient can also be defined in terms similar to those in Equation 6 by introducing a parameter which is called the warhead ballistic factor, W_{wh}/id^2.

$$C = \left(\frac{W_{wh}}{id^2}\right)\left(\frac{PWF}{\gamma_B \times PWF - \gamma_B + 1.0}\right) .$$

D. Range Tables

The BRL range tables have been fitted by an equation of the form

$$V_{req'd} = k_1 + \frac{k_2}{C} . \tag{9}$$

Values of the constants k_1 and k_2 for Q.E. = 45° are given in Table I for ranges up to 40 kilometers. Equation 9 does not accurately describe the range tables for greater ranges.

Table I

LIST OF CONSTANTS FOR CURVE FIT OF RANGE TABLES

Range	k_1	k_2
10 km ($C \leq 2.0$)	650	1,700
10 km ($C > 2.0$)	1,062.5	875
20 km ($C \leq 2.0$)	700	4,500
20 km ($C > 2.0$)	1,387.5	3,125
30 km ($C \leq 2.0$)	1,000	6,200
30 km ($C > 2.0$)	1,712.5	4,775
40 km ($C \leq 2.0$)	1,220	6,960
40 km ($C > 2.0$)	1,912.5	5,575

E. Length Relationships

The relationship of warhead length, weight, and diameter is given as

$$\frac{W_{wh}}{d^3} = \frac{\pi}{4} \rho_{wh} \left[0.536 \left(\frac{l}{d}\right)_{ogive} + \left(\frac{l}{d}\right)_{cyl}\right] \tag{10}$$

for ogive lengths of 2.5 to 4 calibers. This relationship is plotted in Figure 1 for a 4-caliber tangent ogive nose shape.

The motor length to diameter ratio is

$$\left(\frac{l}{d}\right)_{motor} = \frac{W_{wh}}{d^3} \left[\frac{(Q - 1)(PWF)}{L_D \rho_P \frac{\pi}{4}}\right], \tag{11}$$

where L_D is the volumetric loading density of the motor, or V_{prop}/V_{motor}. ρ_P is the density of the propellant. Figure 2 shows the effect of Q and PWF on the ratio of $(l/d)_{motor}$ to W_{wh}/d^3.

V. METHOD OF COMPUTATION OF GROWTH FACTORS

The method of computation of growth factors is an iteration procedure using Equations 2, 3, 6, 8, and 9. The computations were made on an IBM 1620 digital computer, and the iteration was allowed to run until the burnout velocity given by Equation 2 was within 0.5 per cent of the required velocity given by Equation 9.

VI. PRESENTATION OF GROWTH FACTOR DATA

The missile weight data are presented in Figures 3 through 18 in a dimensionless form (growth factor) in order to present the maximum amount of information on a minimum number of graphs.

VII. VERIFICATION OF ACCURACY OF THE METHOD OF COMPUTATION

Several point mass trajectories have been run to verify the accuracy of the growth factor data. At each of the four ranges considered, two design points were chosen for verification on the point mass deck. Table II gives the results of these runs and a comparison with the values obtained from the method used in this study.

Table II

VERIFICATION OF ACCURACY OF THE METHOD OF COMPUTATION

Range, km (Predicted)	$\frac{W_{wh}}{id^2}$	$V_{burnout}$ ft/sec (Predicted)	$V_{burnout}$ ft/sec (Point mass)	Range, km (Point mass)	Range error, percent
10	0.75	2,359	2,368	10.44	4.4
10	1.5	1,596	1,631	10.74	7.4
20	0.75	3,889	3,944	21.74	8.7
20	1.5	2,809	2,879	21.36	6.8
30	0.75	4,536	4,628	32.68	8.9
30	1.5	3,557	3,657	33.21	10.7
40	0.75	4,789	4,899	43.13	7.8
40	1.5	3,884	3,997	44.20	10.5

VIII. ROCKET LENGTHS

Because there are so many parameters affecting the length of a rocket (range, warhead weight, diameter, specific impulse, propellant weight fraction, motor volumetric loading fraction, and propellant density), it is not practical to present parametric length data. The relationships presented in Section IV. E., however, will aid in the estimation of the length of a rocket with a given set of performance parameters. A sample problem will best illustrate the method of estimating the rocket length. Assume the following parameters:

Range	30 km
Warhead weight	200 lb
Diameter	10 in.
PWF	0.5
I_{sp}	220 sec
i (drag form factor)	1.0
Warhead density	0.055 lb/in.^{3}
Propellant density	0.060 lb/in.^{3}
L_D	0.5
$(l/d)_{ogive}$	4

1. Compute warhead density parameter.

$$\frac{W_{wh}}{d^3} = \frac{200}{10^3} = 0.20 \ .$$

2. Obtain length of cylindrical portion of warhead from curve.

$$\left(\frac{l}{d}\right)_{cyl} = 2.5 \text{ (From Fig. 1).}$$

3. Compute warhead ballistic factor.

$$\frac{W_{wh}}{id^2} = \frac{200}{(1.0)(10^2)} = 2.00 \ .$$

4. Obtain growth factor from curve.

$$Q = 3.20 \text{ (From Fig. 12).}$$

5. Compute rocket takeoff weight.

$$W_{to} = Q \times W_{wh} = 3.20 \times 200 = 640 \text{ lb}$$

6. Obtain motor length parameter from curve.

$$\frac{\left(\frac{l}{d}\right)_{motor}}{\frac{W_{wh}}{d^3}} = 58 \text{ (From Fig. 2)}.$$

7. Compute motor length in calibers.

$$\left(\frac{l}{d}\right)_{motor} = 58 \times 0.2 = 11.6$$

7. Compute rocket length in calibers.

$$\left(\frac{l}{d}\right)_{rkt} = \left(\frac{l}{d}\right)_{ogive} + \left(\frac{l}{d}\right)_{cyl} + \left(\frac{l}{d}\right)_{motor} = 4.0 + 2.5 + 11.6 = 18.1.$$

9. Compute rocket length in inches.

$$l_{rkt} = 18.1 \times 10 = 181.$$

IX. LENGTH-DIAMETER TRADEOFFS

In the absence of diameter restraints, such as minimum diameter of warhead devices, the selection of the optimum diameter for a given rocket mission is the result of tradeoffs between the length and weight of the rocket. For fin stabilized rockets, the range of length to diameter ratios usually considered runs from 8 to 20, with the lower restraint imposed because of stabilization considerations, and the upper limit imposed for structural reasons.

Figures 19, 20, and 21 show length and weight tradeoffs for ranges of 20, 30, and 40 kilometers, based on a PWF of 0.65 and an I_{sp} of 240 seconds. Similar tradeoffs can be made for other values of propulsion efficiency using data presented in this report. These figures show the relationship between length and weight of rocket for various fixed warhead weights. It can be seen that there is an asymptote at each end of the constant payload curves, showing that there is a minimum rocket weight regardless of how much the diameter is reduced. There is also a minimum length, regardless of how much the diameter is increased.

The selection of the optimum diameter for a given payload weight must therefore be based on consideration of the following factors:

1. Rocket length to diameter ratio.

2. Maximum allowable length.

3. Maximum allowable weight.

4. Special warhead considerations limiting the diameter.

5. Storage.

6. Boost acceleration requirements (acceleration is inversely proportioned to motor l/d because of interior ballistic limitations).

7. Accuracy requirements (free flight dispersion due to meteorological effects are reduced by high ballistic coefficients, which correspond to long, slender configurations).

X. LAUNCH ANGLE TRADEOFFS

In certain cases it may be desirable that a system be designed to achieve a certain range at a launch angle less than the optimum, in order to reduce time of flight and to decrease the sensitiveness of the rocket to meteorological effects. Figure 22 shows the effect on range of launching at 30° Q.E., compared to launching at 45° Q.E., which is near optimum. Since this figure only shows the range reduction at the lower Q.E., it is necessary to know the relationship between range and growth factor to compute the effect of Q.E. on growth factor required to achieve a given range. Fortunately, the relationship between range and growth factor is nearly linear, as illustrated by Figure 23. As a first approximation, the percentage increase in growth factor required to achieve a given range at a Q.E. other than 45° will be equal to the percentage range reduction given in Figure 22 for that Q.E. An example will illustrate how this correction can be made. Assume that it is desired to estimate the weight of a rocket with a range of 30 kilometers at 30° Q.E. This rocket has the following characteristics:

W_{wh} = 150 lb

Diameter = 10 in.

$PWF = 0.5$

$I_{sp} = 220$

i (drag form factor) = 1.0

1. Compute warhead ballistic factor.

$$\frac{W_{wh}}{id^2} = \frac{150}{(1.0)(10^2)} = 1.5 \text{ lb/in.}^2$$

2. Read growth factor required for 45° Q.E. from Figure 12.

$$Q_{45°} = 3.70$$

3. Compute rocket weight at takeoff and burnout.

$$W_{to} = Q \times W_{wh} = (3.70)(150) = 555 \text{ lb}$$

$$W_{bo} = W_{to} - PWF\,(W_{to} - W_{wh}) = 555 - 0.5(555 - 150)$$

$$= 347.5 \text{ lb}$$

4. Compute ballistic coefficient.

$$\frac{W_{bo}}{id^2} = \frac{347.5}{(1.0)(10^2)} = 3.475$$

5. Compute burnout velocity.

$$V_{B\ actual} = 0.95\ I_{sp}\ g \ln (\gamma_B) = 0.95(220)(32.2) \ln \frac{555}{347.5}$$

$$= 3{,}150 \text{ ft/sec}$$

6. Obtain range reduction for 30° firing compared to 45° firing from Figure 22.

$$\frac{R_{30}}{R_{45}} = 0.86 \text{ (14\% reduction)}$$

7. Compute increased growth factor (assuming a linear relationship between growth factor and range).

$$Q_{30°} = \frac{Q_{45°}}{0.86} = \frac{3.70}{0.86} = 4.30$$

8. Compute rocket weight required for range of 30 kilometers at 30° Q.E.

$$W_{to} = Q \times W_{wh} = 4.3 \times 150 = 645 \text{ lb}$$

XI. CONCLUSIONS

The data presented in this report will permit the rapid estimation of rocket weights and lengths for given sets of performance requirements. The data are especially useful in the determination of weight-length-diameter tradeoffs, and the assessment of the relative importance of the propulsion efficiency parameters, I_{sp} and PWF. This report is intended for use in making "first-cut" approximations only, and should be followed by more detailed studies on each design point selected.

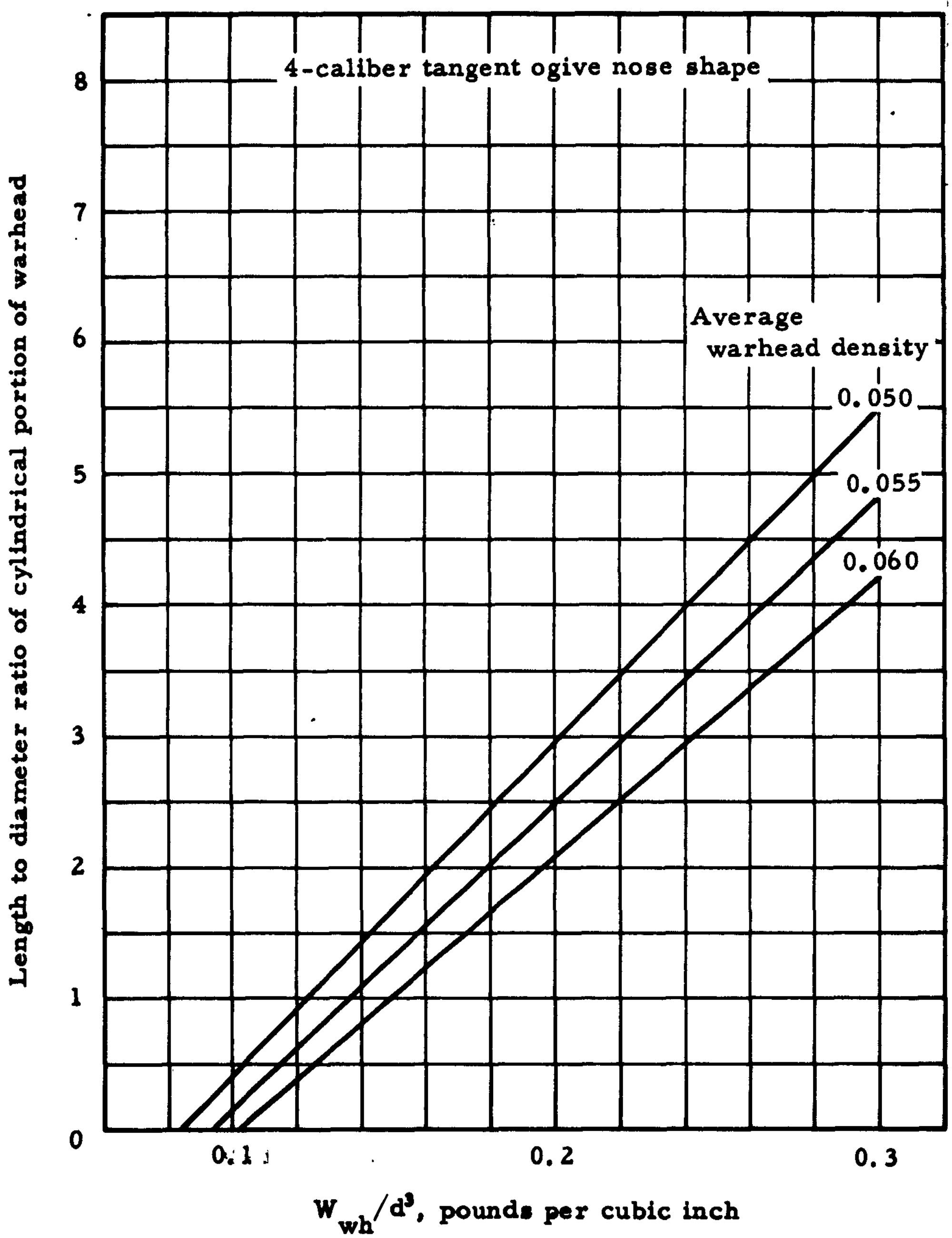

Figure 1. WARHEAD CYLINDRICAL LENGTH VERSUS W_{wh}/d^3 FOR VARIOUS DENSITIES.

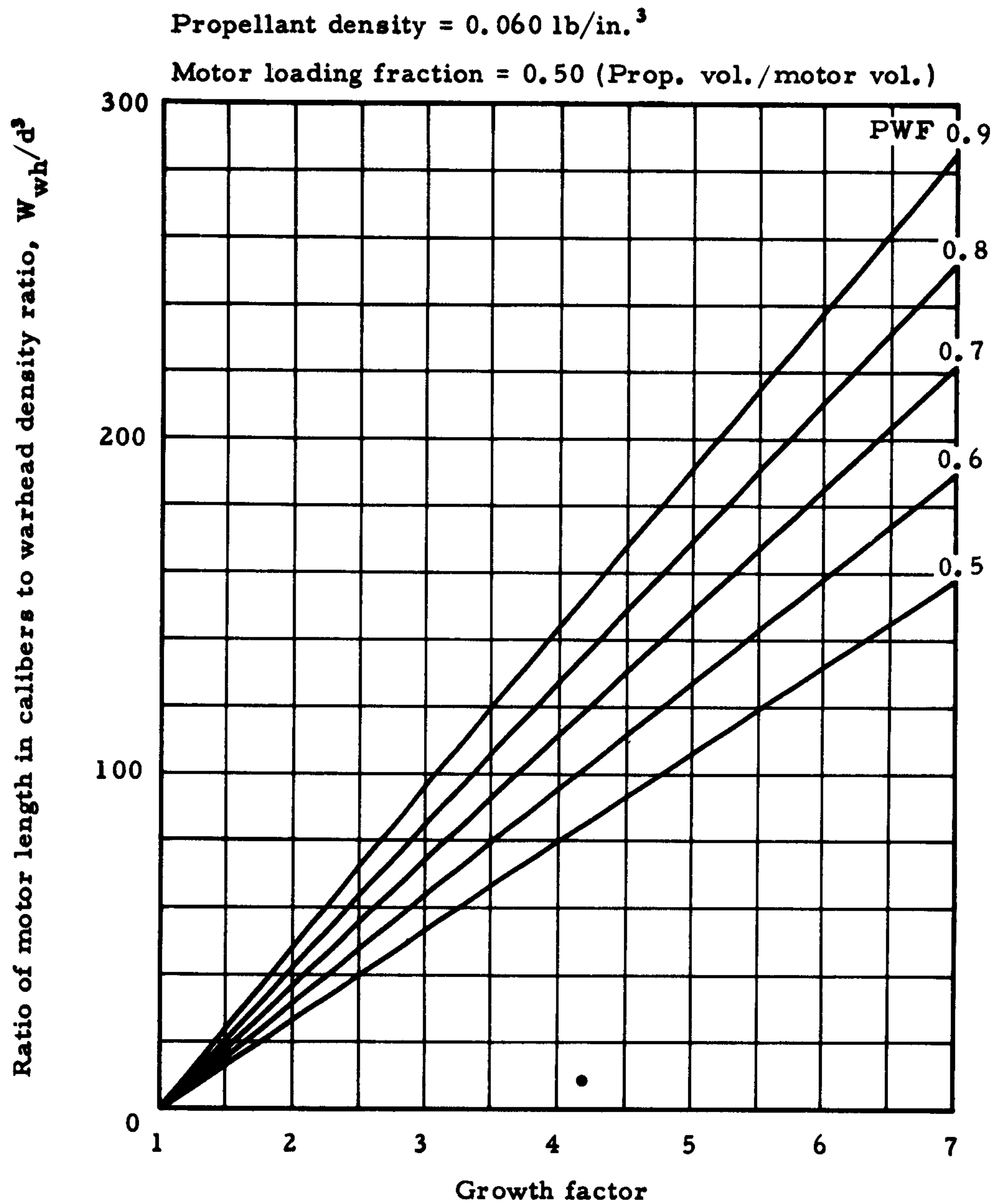

Figure 2. MOTOR LENGTH PARAMETER VERSUS GROWTH FACTOR

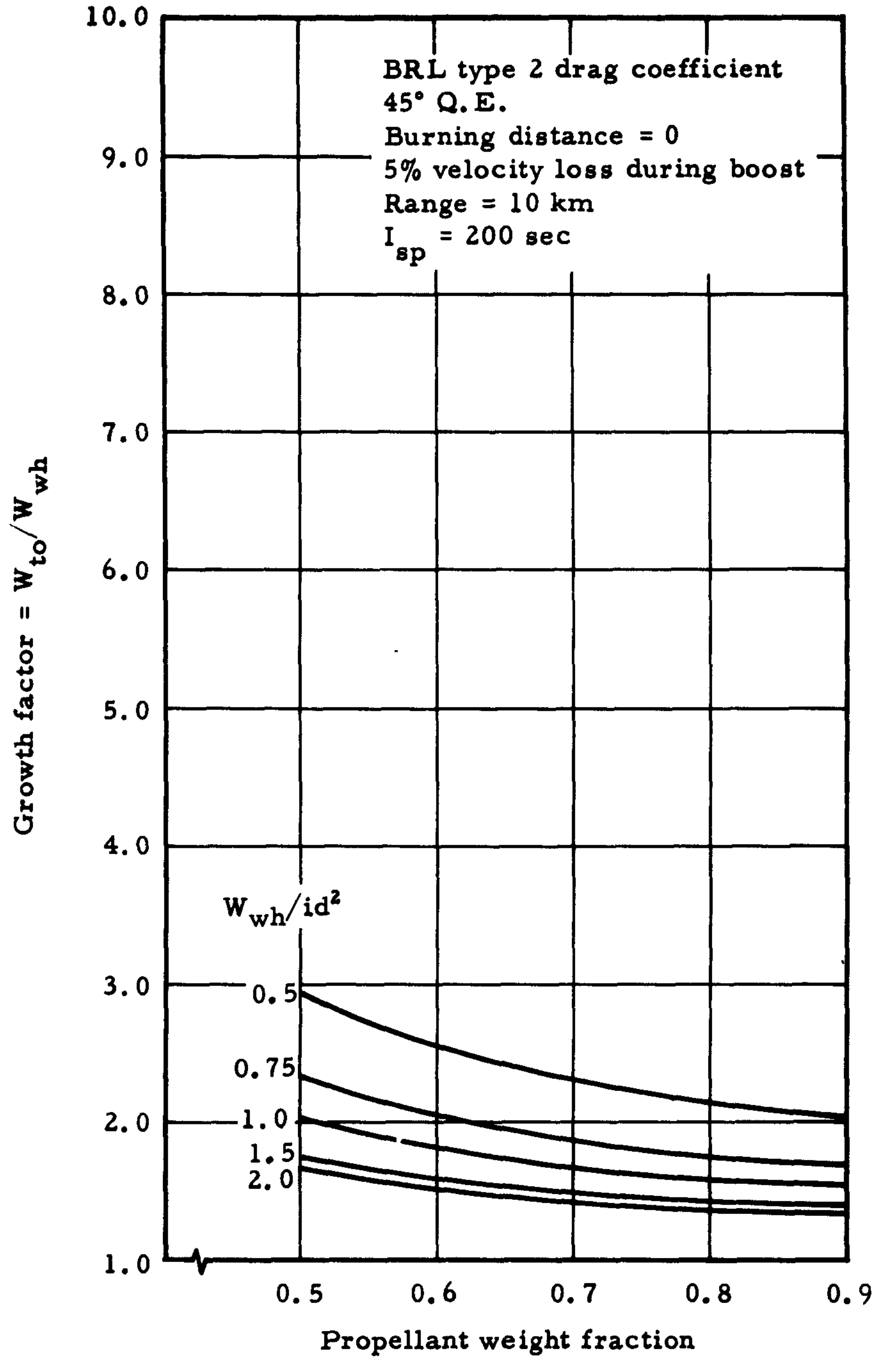

Figure 3. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

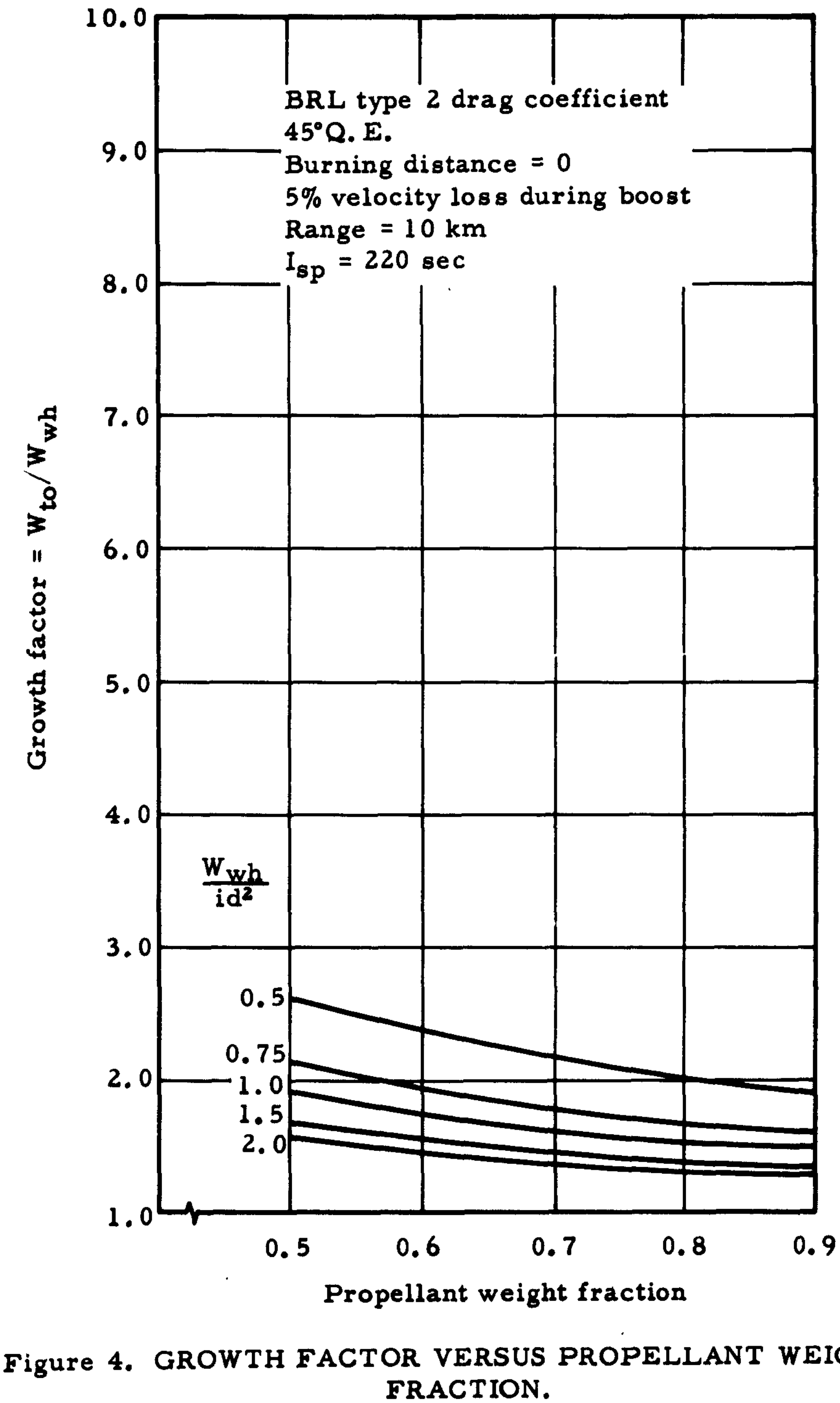

Figure 4. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

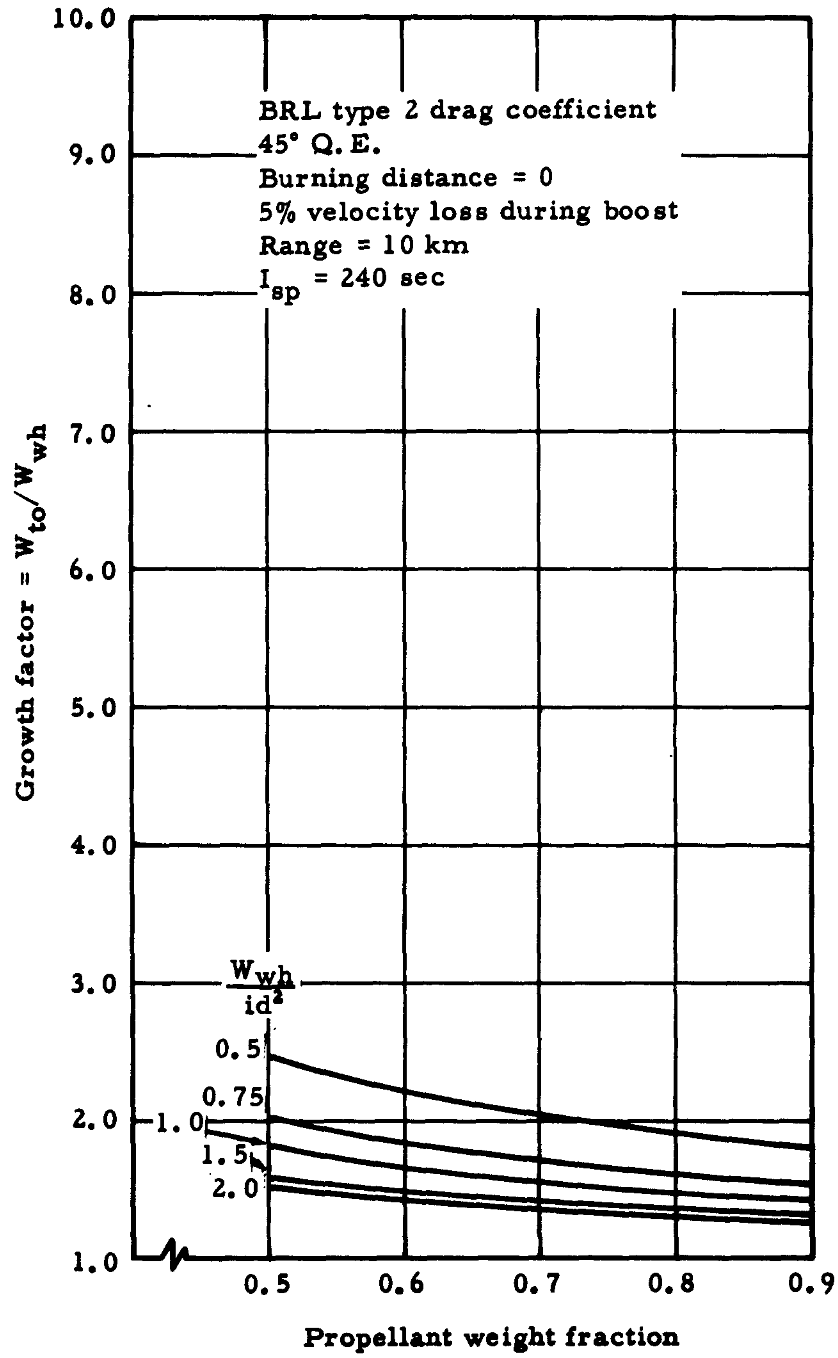

Figure 5. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

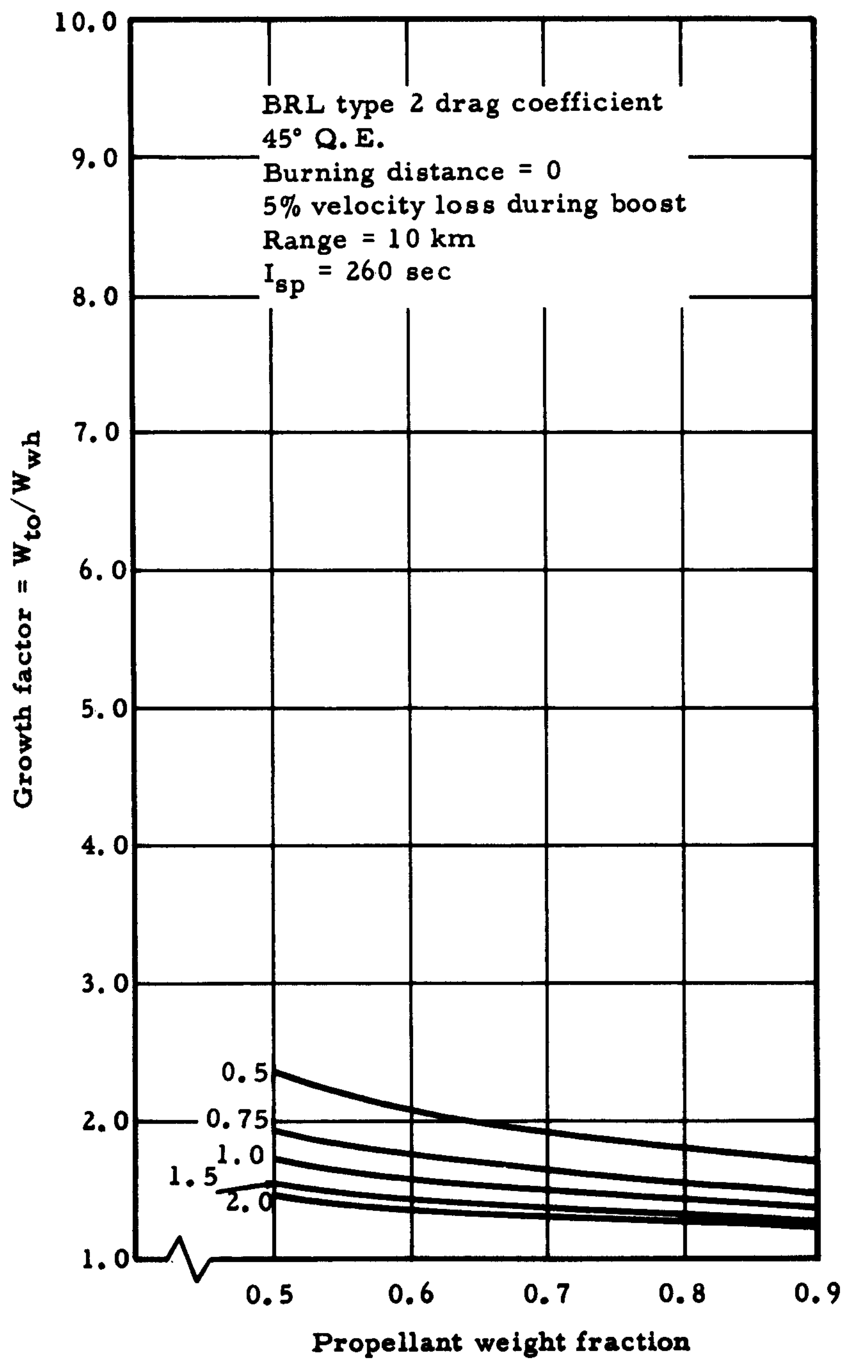

Figure 6. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

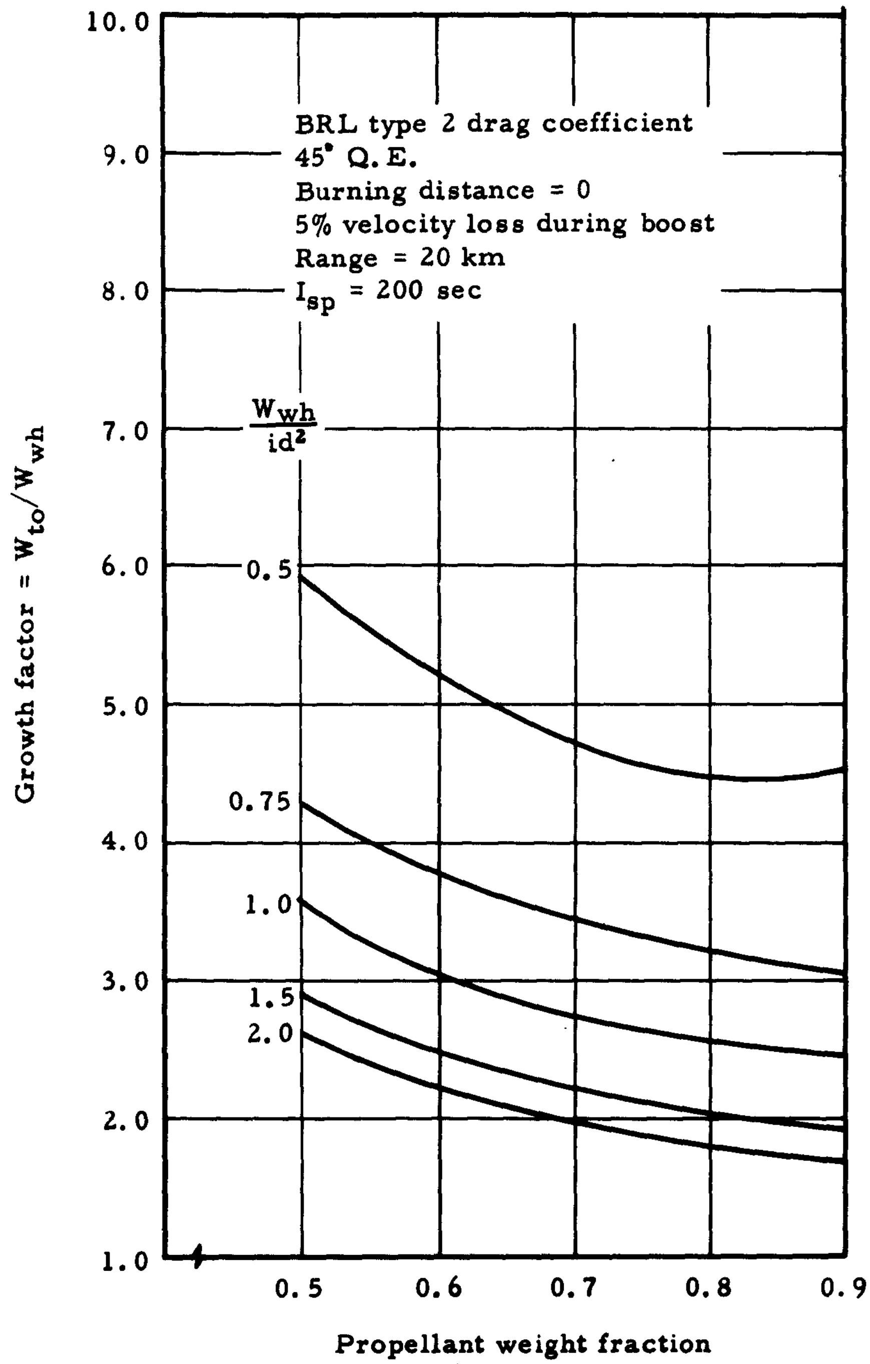

Figure 7. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

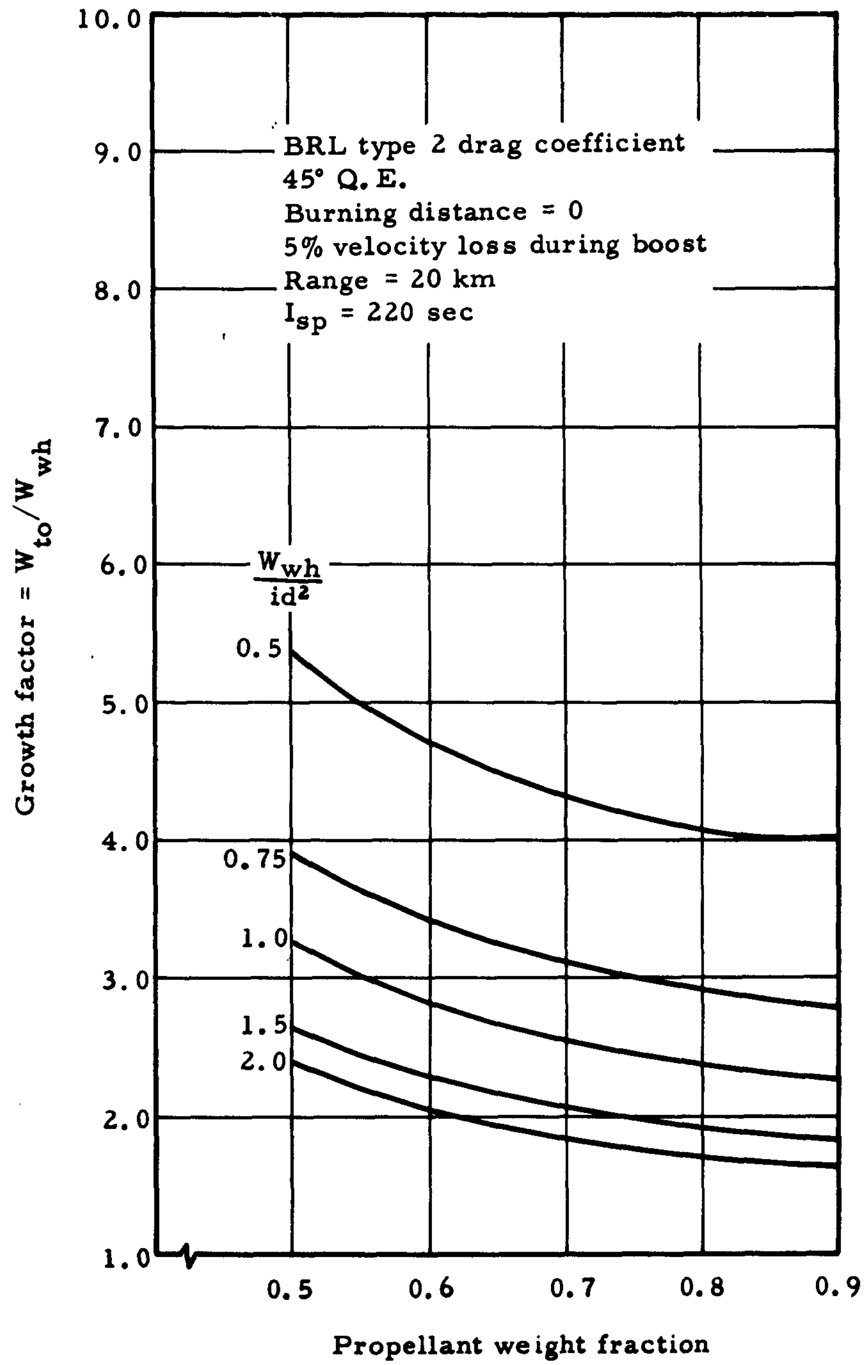

Figure 8. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

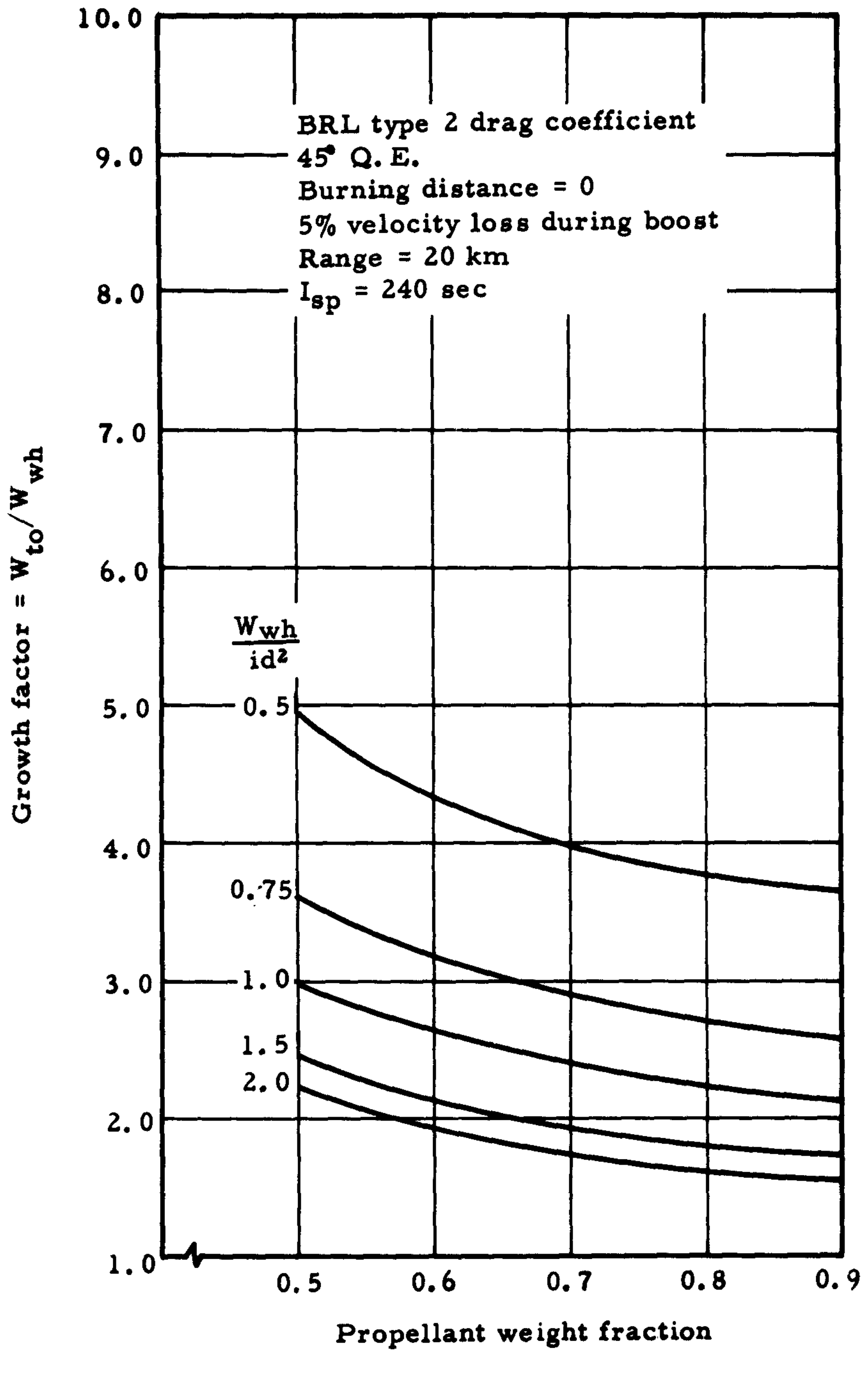

Figure 9. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

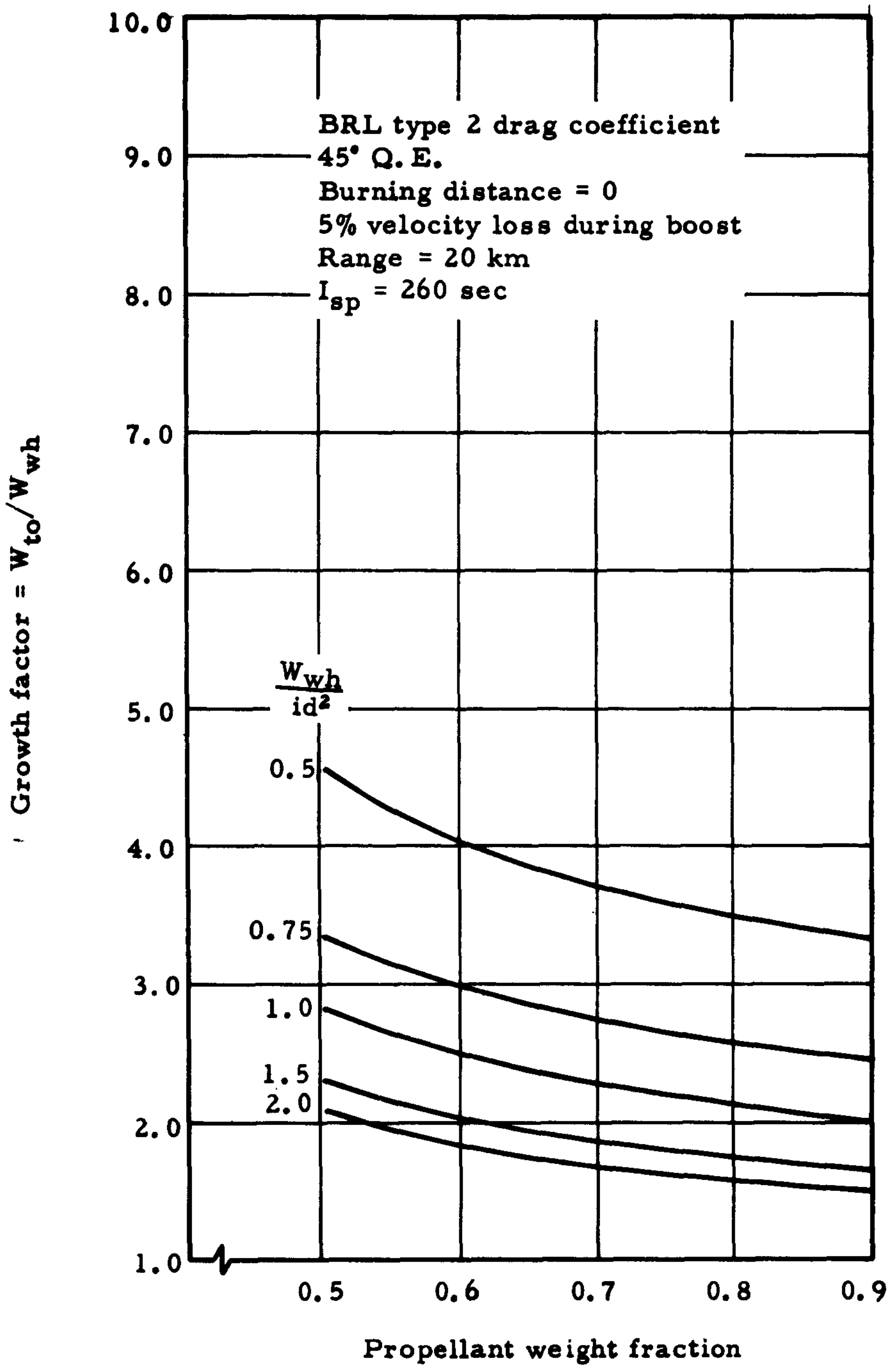

Figure 10. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

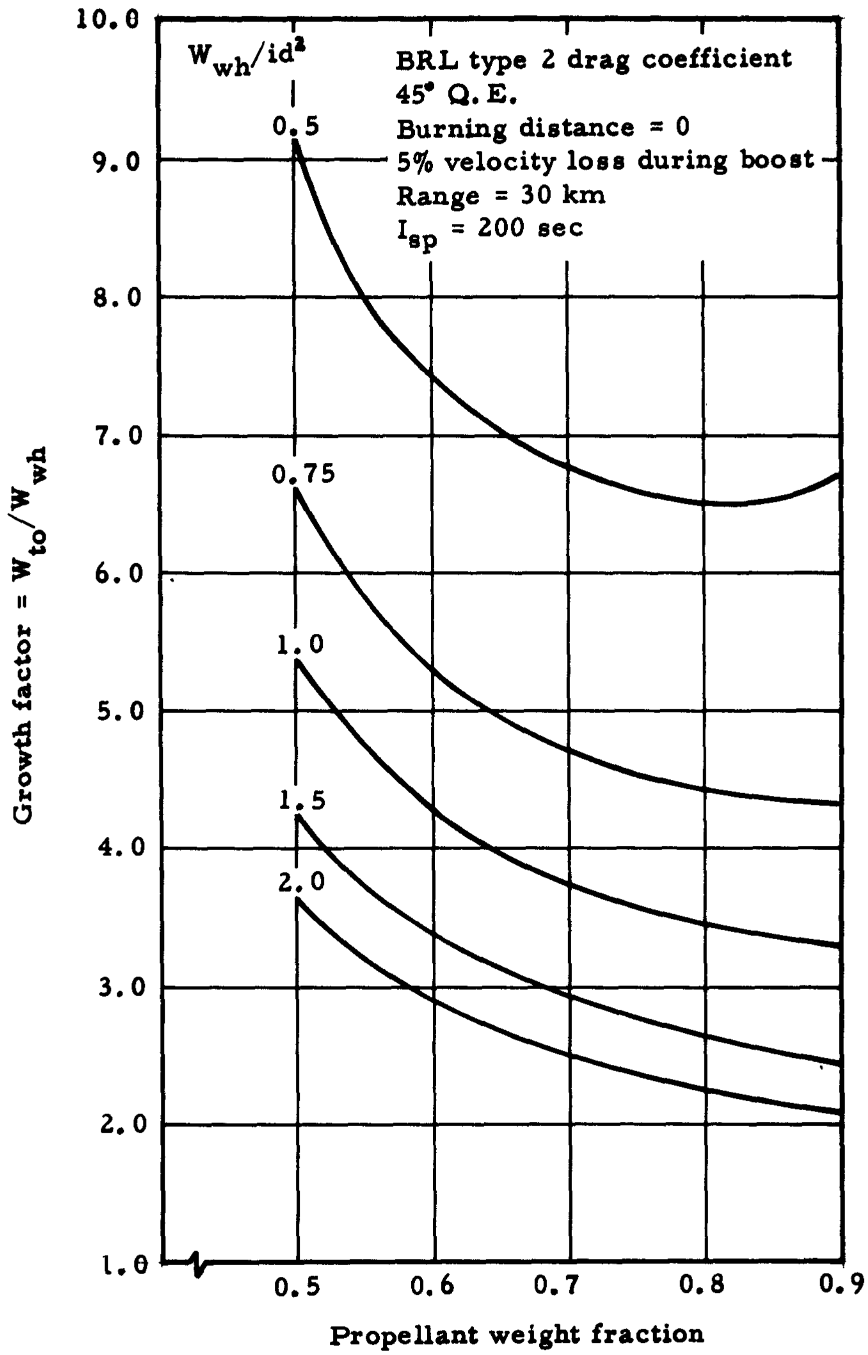

Figure 11. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

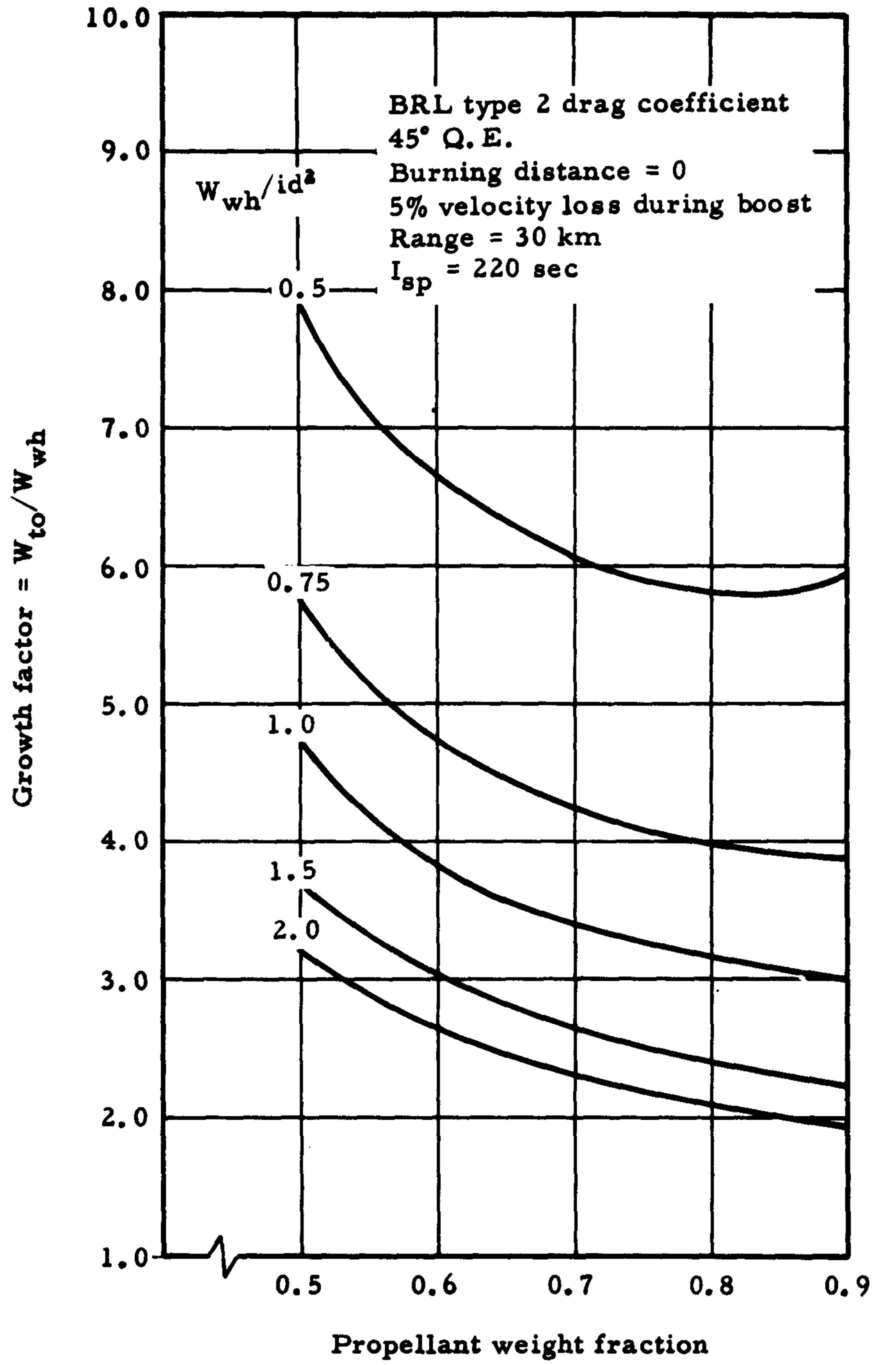

Figure 12. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

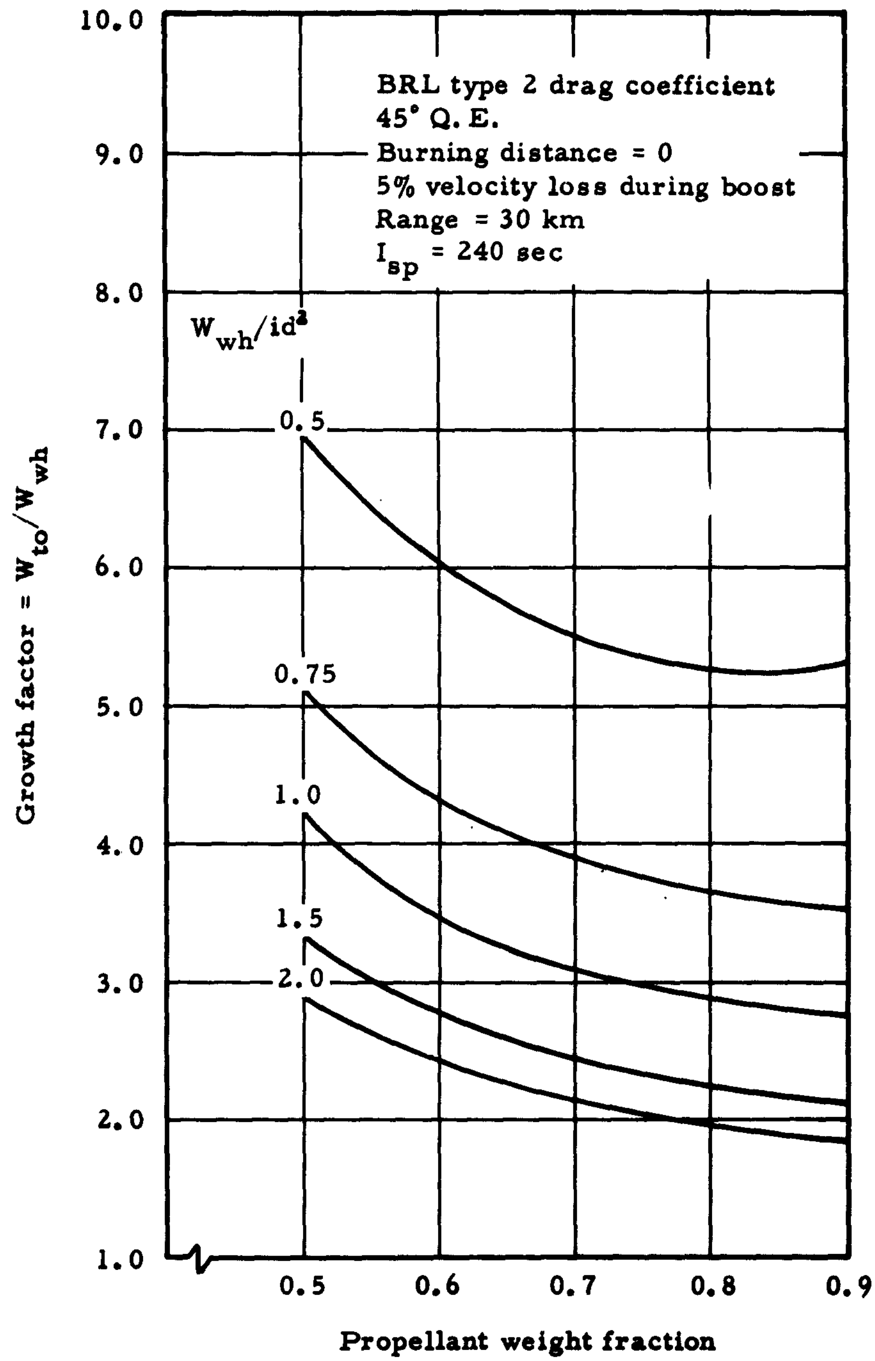

Figure 13. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

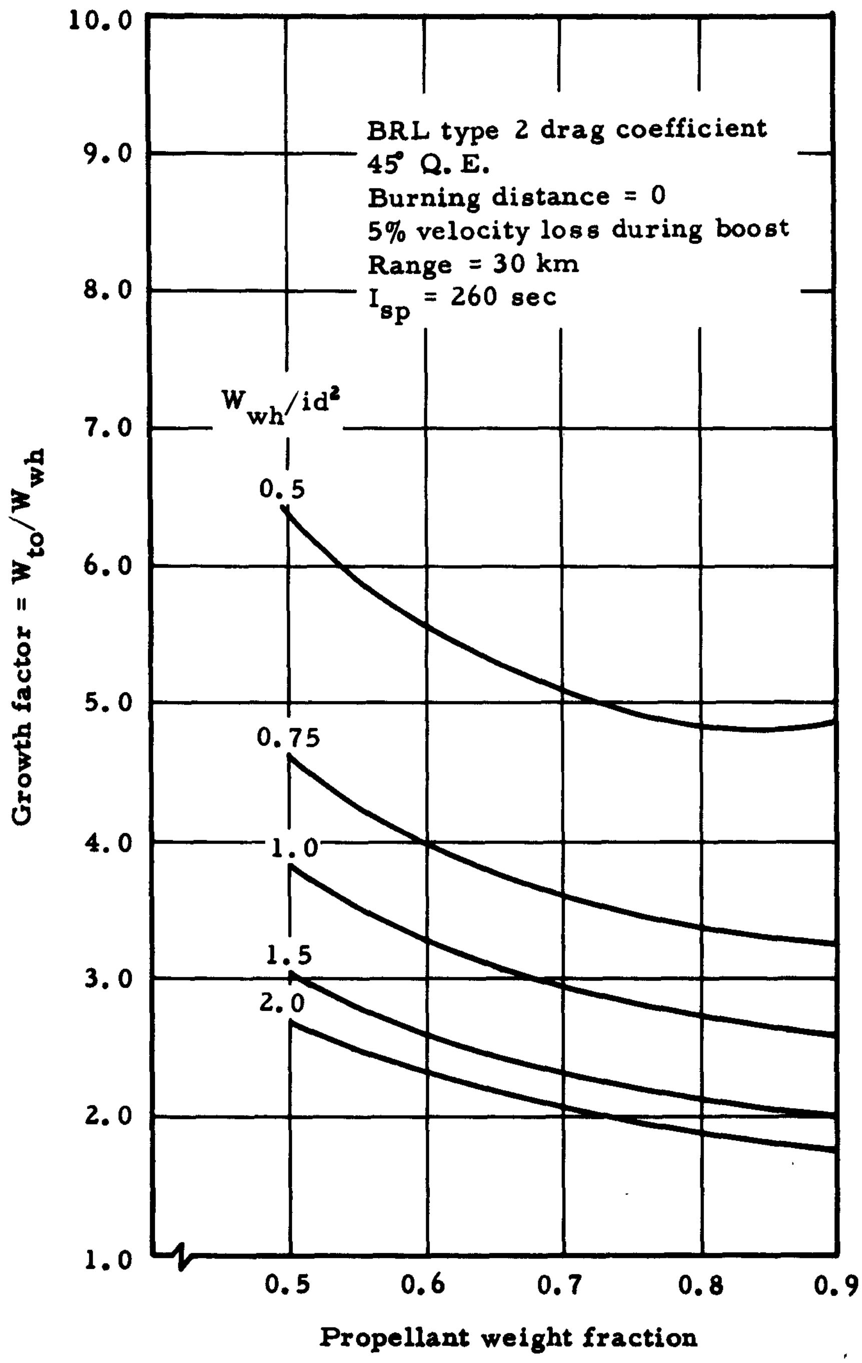

Figure 14. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

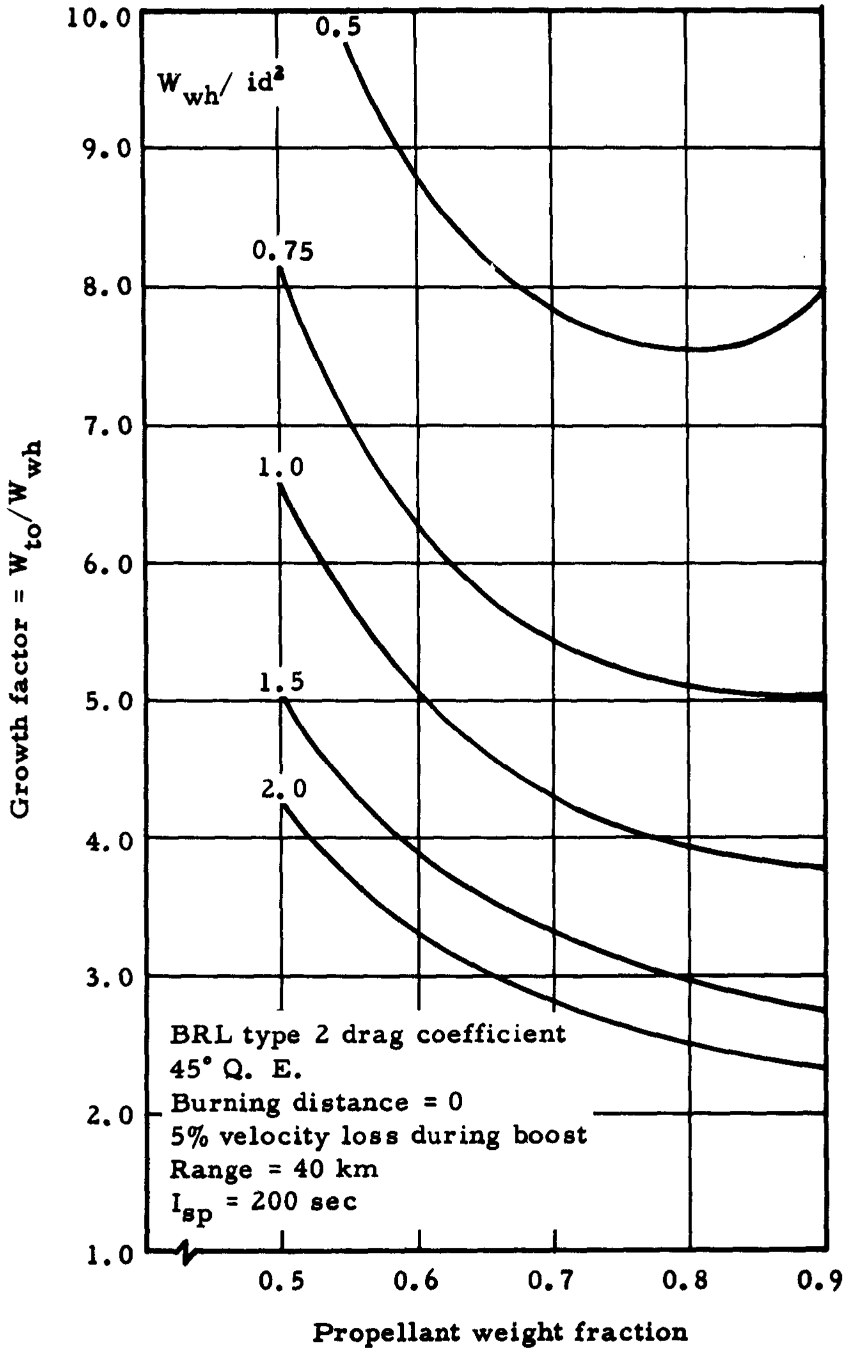

Figure 15. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

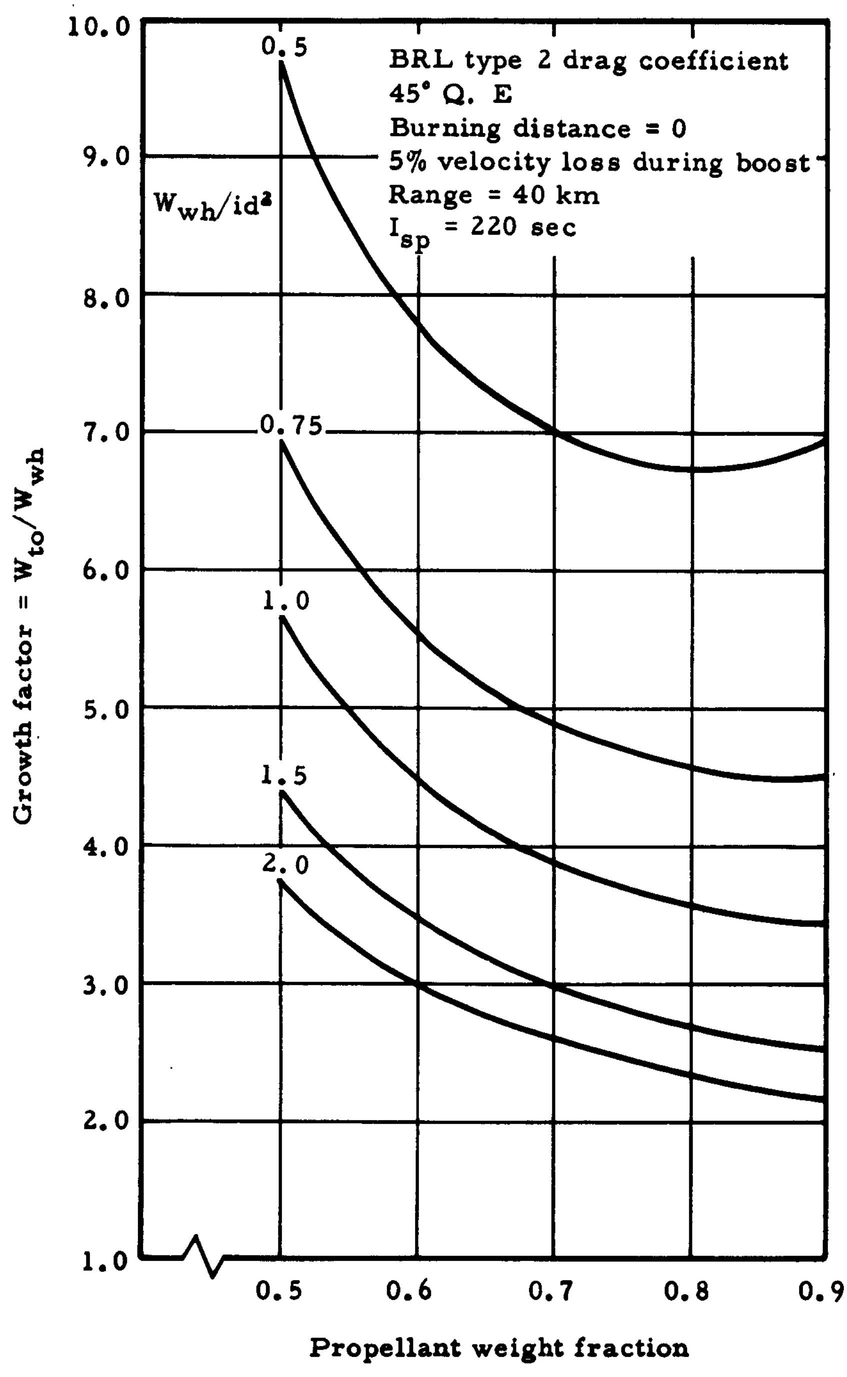

Figure 16. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

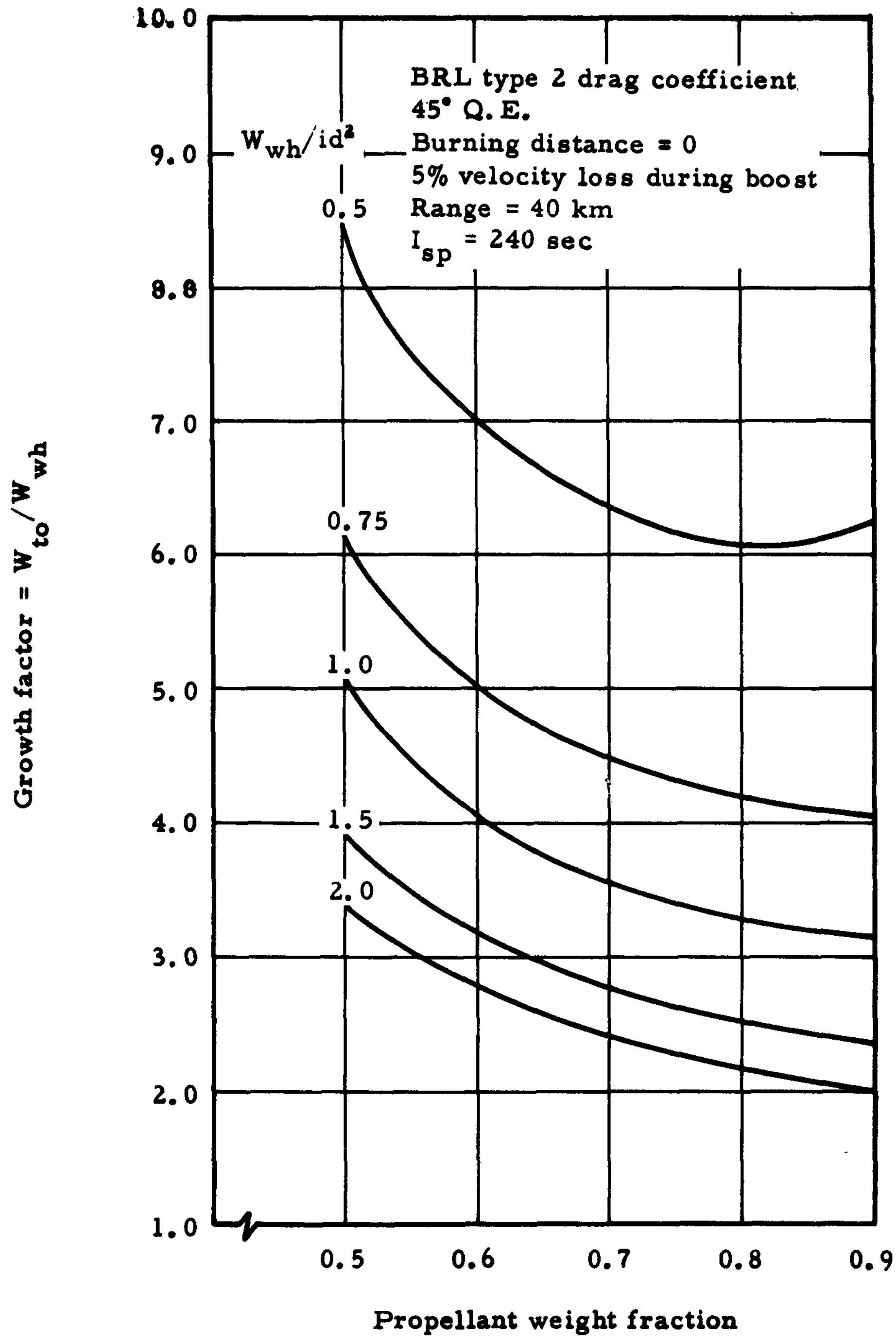

Figure 17. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

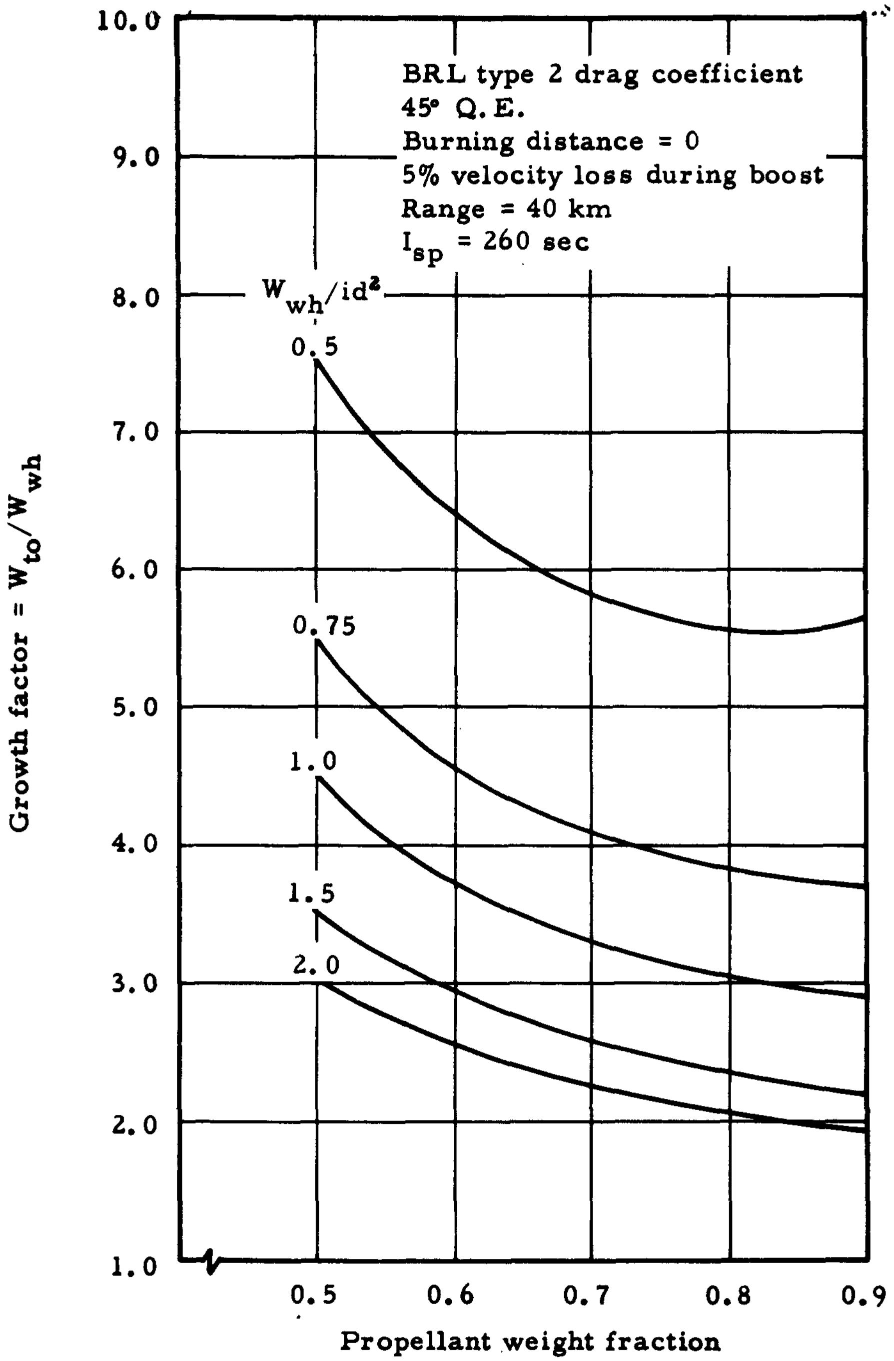

Figure 18. GROWTH FACTOR VERSUS PROPELLANT WEIGHT FRACTION.

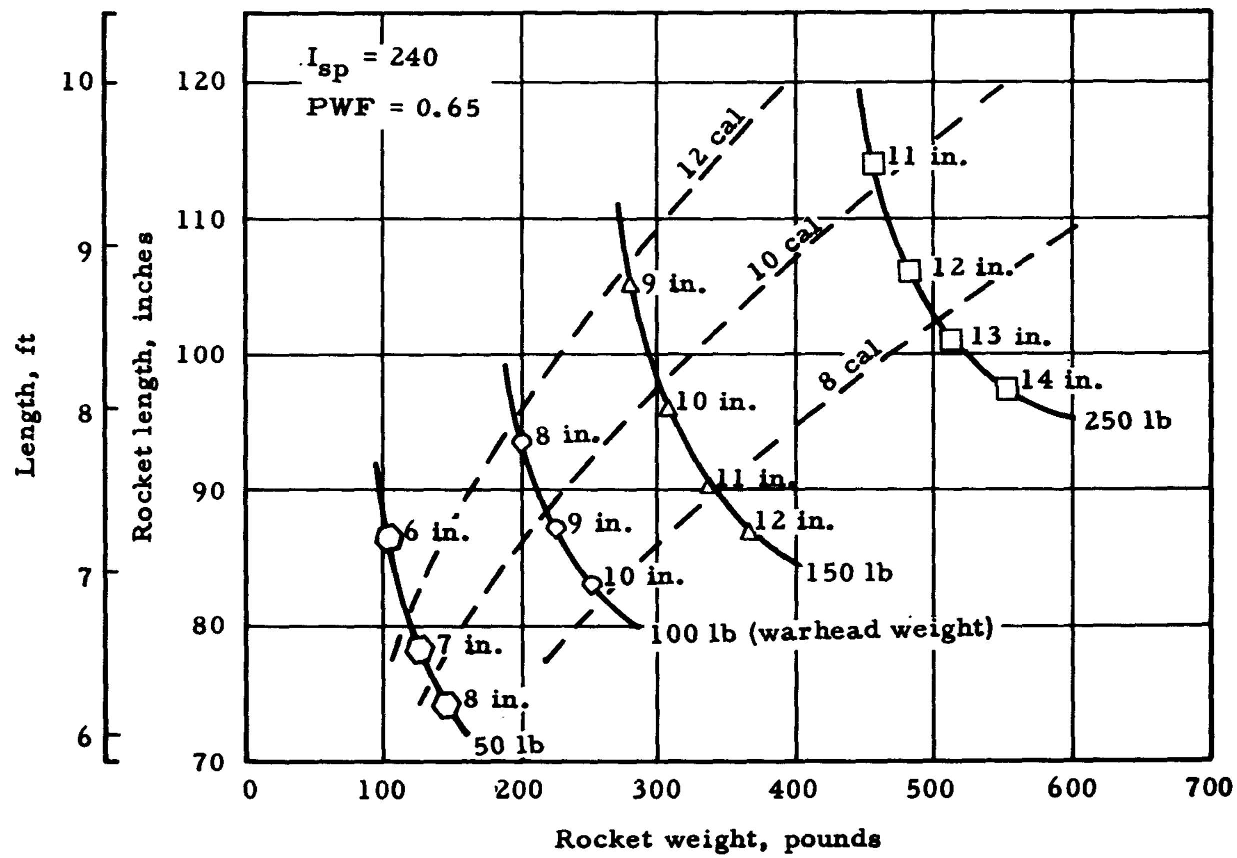

Figure 19. WEIGHT AND LENGTH TRADEOFFS FOR RANGE = 20 KM.

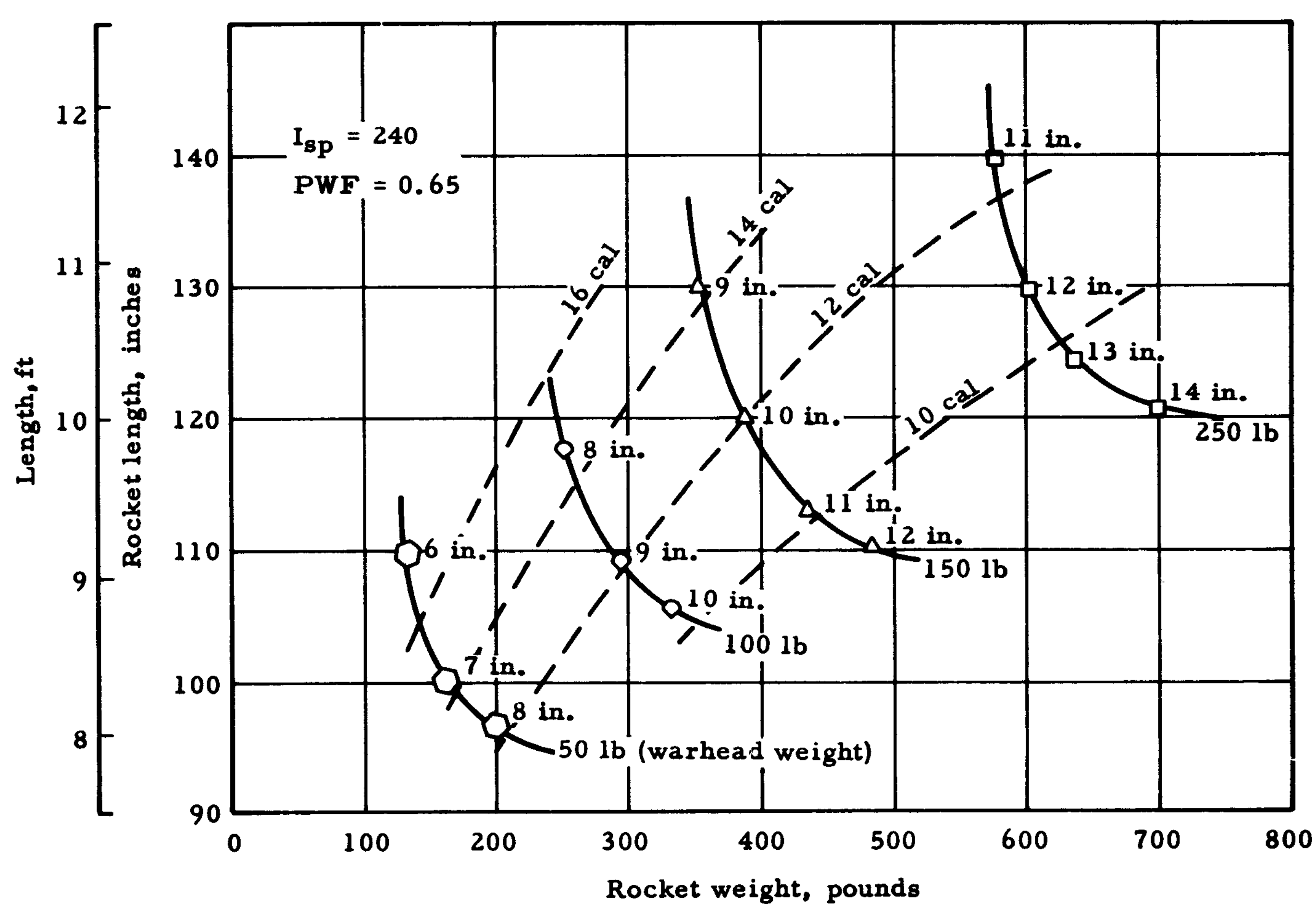

Figure 20. WEIGHT AND LENGTH TRADEOFFS FOR RANGE = 30 KM.

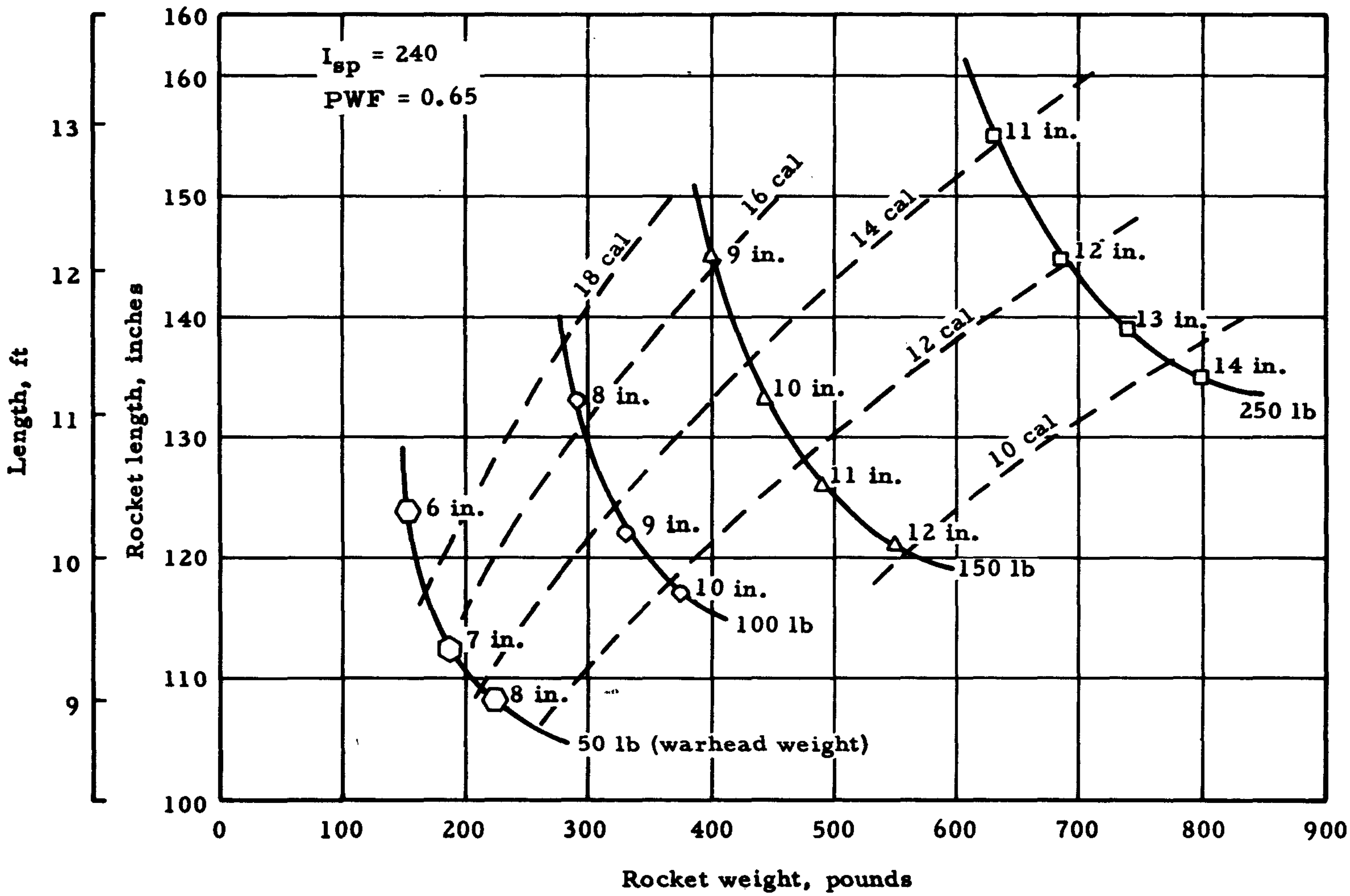

Figure 21. WEIGHT AND LENGTH TRADEOFFS FOR RANGE = 40 KM.

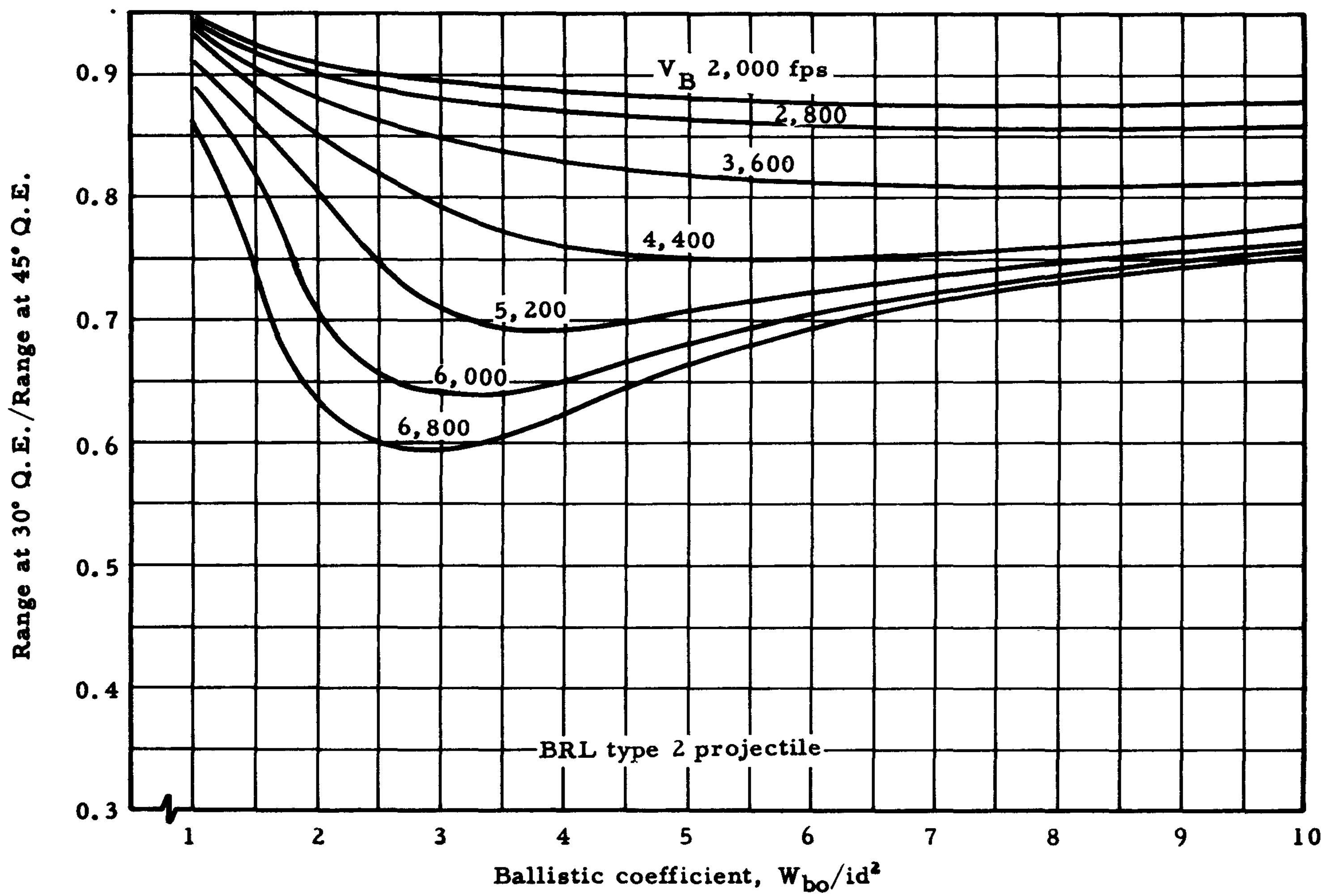

Figure 22. COMPARISON OF RANGE AT 30° Q.E. TO RANGE AT 40° Q.E.

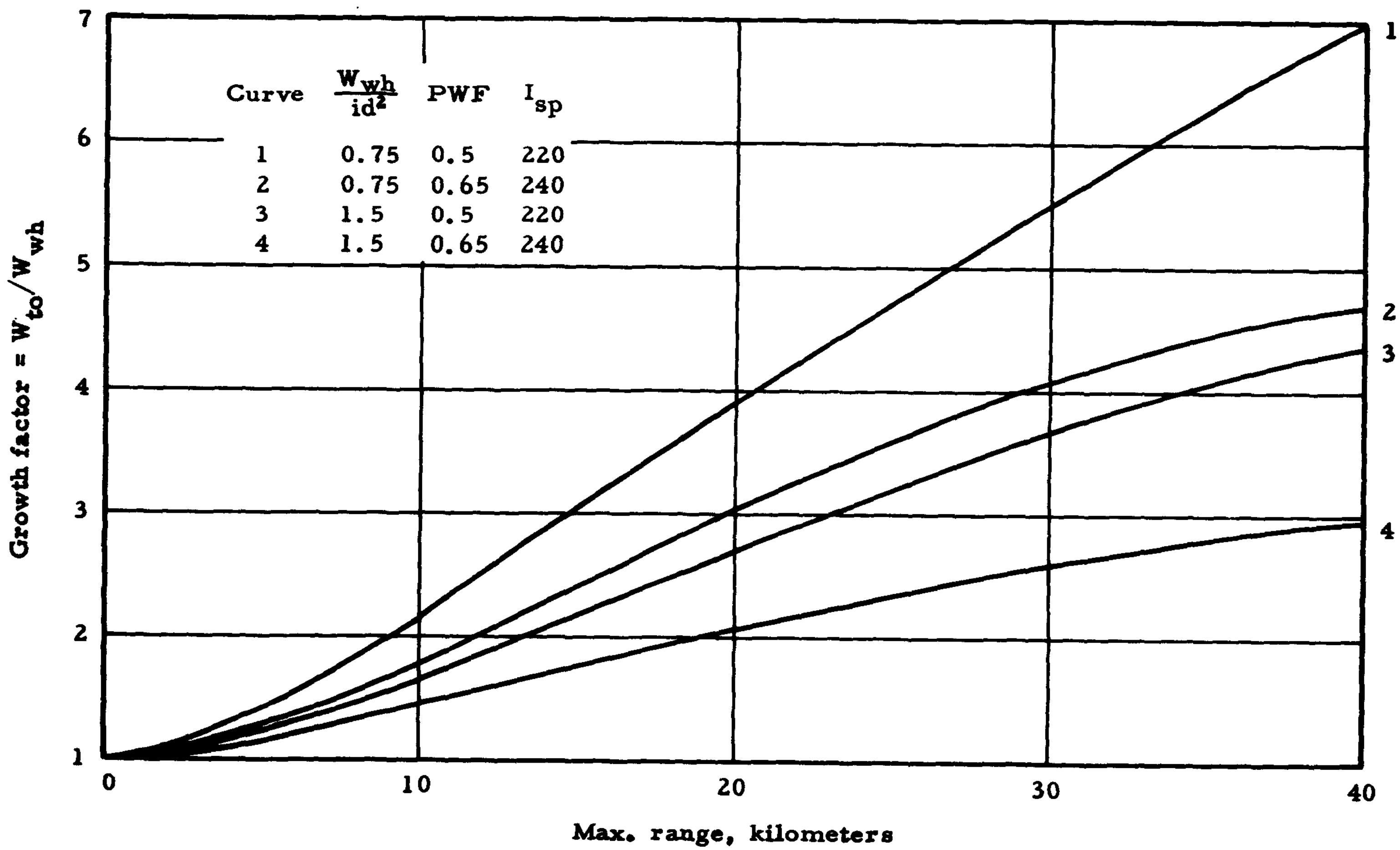

Figure 23. EFFECT OF RANGE ON GROWTH FACTOR.

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2. Rockets--Design
3. Rockets--Exterior ballistics
4. Rockets--Range
5. Rockets--Specific impulse
6. Rockets--Weight

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PARAMETRIC DESIGN CURVES FOR SHORT RANGE, HIGH ACCELERATION BALLISTIC ROCKETS (U) - Herman R. Oswell

Army Msl Cmd RF-TR-63-7, 1 Apr 63, 35 pp - illus. Unclassified Report

This report is a compilation of design curves which will permit the rapid estimation of the effects of range, warhead weight, diameter, specific impulse, and propellant weight fraction on the weight and length of short range, high acceleration ballistic rockets.

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1. Exterior ballistics--Mathematical analysis
2. Rockets--Design
3. Rockets--Exterior ballistics
4. Rockets--Range
5. Rockets--Specific impulse
6. Rockets--Weight

I. Oswell, Herman R.

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